MSE 7025
Magnetic Materials
(and Spintronics)

Lecture 5: Interactions, Episode I

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Course Outline

- **Time Table**

<table>
<thead>
<tr>
<th>Week</th>
<th>Date</th>
<th>Lecture</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Feb 24</td>
<td>Introduction</td>
</tr>
<tr>
<td>2</td>
<td>March 2</td>
<td>Magnetic units and basic E&amp;M</td>
</tr>
<tr>
<td>3</td>
<td>March 9</td>
<td>Magnetization: From classical to quantum</td>
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<tr>
<td>4</td>
<td>March 16</td>
<td>No class (APS March Meeting, Baltimore)</td>
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<tr>
<td>5</td>
<td>March 23</td>
<td>Category of magnetism</td>
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<tr>
<td>6</td>
<td>March 30</td>
<td>From atom to atoms: Interactions I (oxides)</td>
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<tr>
<td>7</td>
<td>April 6</td>
<td>From atom to atoms: Interactions II (metals)</td>
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<tr>
<td>8</td>
<td>April 13</td>
<td>Magnetic anisotropy</td>
</tr>
<tr>
<td>9</td>
<td>April 20</td>
<td>Mid-term exam</td>
</tr>
<tr>
<td>10</td>
<td>April 27</td>
<td>Domain and domain walls</td>
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</table>
# Course Outline

## Time Table

<table>
<thead>
<tr>
<th>Week</th>
<th>Date</th>
<th>Lecture</th>
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<tbody>
<tr>
<td>11</td>
<td>May 4</td>
<td>Magnetization process (SW or Kondorsky)</td>
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<td>12</td>
<td>May 11</td>
<td>Characterization: VSM, MOKE</td>
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<tr>
<td>13</td>
<td>May 18</td>
<td>Characterization: FMR</td>
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<tr>
<td>14</td>
<td>May 25</td>
<td>Transport measurements in materials I: Hall effect</td>
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<tr>
<td>15</td>
<td>June 1</td>
<td>Transport measurements in materials II: MR</td>
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<tr>
<td>16</td>
<td>June 8</td>
<td>MRAM: TMR and spin transfer torque</td>
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<tr>
<td>17</td>
<td>June 15</td>
<td>Guest lecture (TBA)</td>
</tr>
<tr>
<td>18</td>
<td>June 22</td>
<td>Final exam</td>
</tr>
</tbody>
</table>
Ferromagnetism

- Curie-Weiss law
  - Weiss internal field, Weiss exchange field, Weiss molecular field

\[
H_E = \lambda M
\]

\( \lambda \approx 2000 \)

(Remember Hw2?)

\[
M = \chi H = \chi (H_{\text{applied}} + H_E) = \chi (H_{\text{applied}} + \lambda M)
\]

\[
\Rightarrow \frac{C}{T} = \frac{M}{H_{\text{applied}} + \lambda M}
\]

\[
\Rightarrow \chi = \frac{M}{H_{\text{applied}}} = \frac{C}{T - \lambda C} = \frac{C}{T - T_c}
\]
Ferromagnetism

- Curie-Weiss law
  - Weiss internal field, Weiss exchange field, Weiss molecular field

\[ H_E = \lambda M \quad \text{\(\lambda \approx 2000\)} \]

(Remember Hw2?)

- Connection to “exchange interaction” and total Hamiltonian

\[
\hat{H} = -\sum_{i,j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + g \mu_B \sum_i \mathbf{S}_i \cdot \mathbf{B} = g \mu_B \sum_i \mathbf{S}_i \cdot (\mathbf{B}_{mf} + \mathbf{B})
\]

\[
\mathbf{B} + \mathbf{B}_{mf} = \mu_0 \left( \mathbf{H} + H_E \right) = \mu_0 \left( \mathbf{H} + \lambda \mathbf{M} \right)
\]
Ferromagnetism

• Curie-Weiss law
  – Weiss internal field, Weiss exchange field, Weiss molecular field

\[
\chi = \frac{M}{H_{\text{applied}}} = \frac{C}{T - \lambda C} = \frac{C}{T - T_c}
\]
Ferromagnetism

• Curie-Weiss law
  – Weiss internal field, Weiss exchange field, Weiss molecular field

\[ \chi = \frac{C}{T - \lambda C} = \frac{C}{T - T_c} \]

• Curie Temperature \( T_c \)

\[ T_c = \lambda C = \frac{\lambda n_v \mu_0 \mu_{\text{eff}}^2}{3k_B} \]

(From \( T_c \) and \( \mu_{\text{eff}} \) we can estimate \( \lambda \), Hw.3)
Ferromagnetism

- Curie-Weiss law
  - Weiss internal field, Weiss exchange field, Weiss molecular field
  - Again, borrow from theory of quantum paramagnetism

\[
M = n_v g \mu_B j \cdot B_j (x) = M_s B_j (x) \approx M_s \tanh (x)
\]

\[
M_s \equiv n_v g \mu_B j
\]

\[
x \equiv g \mu_B j B / k_B T = g \mu_B j \mu_0 \left( H_{\text{applied}} + \lambda M \right) / k_B T
\]

\[
M = \frac{1}{\lambda} \left( \frac{k_B T}{g \mu_B j \mu_0} x - H_{\text{applied}} \right) \approx \frac{1}{\lambda} \left( \frac{k_B T}{\mu_B \mu_0} x - H_{\text{applied}} \right)
\]

\(j = 1/2\)
Ferromagnetism

- Curie-Weiss law
  - Weiss internal field, Weiss exchange field, Weiss molecular field
  - Again, borrow from theory of quantum paramagnetism
  - Find $M(x)$ by plotting these two functions!

\[
\frac{M}{M_s} = \tanh(x)
\]

\[
\frac{M}{M_s} = \frac{1}{\lambda n_v \mu_B} \left( \frac{k_B T}{\mu_B \mu_0} x - H_{\text{applied}} \right)
\]
Ferromagnetism

• Curie-Weiss law
  – Weiss internal field, Weiss exchange field, Weiss molecular field
  – Again, borrow from theory of quantum paramagnetism
Ferromagnetism

- Curie-Weiss law
  - Curie temperature of various ferromagnetic materials

<table>
<thead>
<tr>
<th>Substance</th>
<th>Room temperature</th>
<th>0 K</th>
<th>(n_p(0 \text{ K})), per formula unit</th>
<th>Curie temperature, in K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe</td>
<td>1707</td>
<td>1740</td>
<td>2.22</td>
<td>1043</td>
</tr>
<tr>
<td>Co</td>
<td>1400</td>
<td>1446</td>
<td>1.72</td>
<td>1388</td>
</tr>
<tr>
<td>Ni</td>
<td>485</td>
<td>510</td>
<td>0.606</td>
<td>627</td>
</tr>
<tr>
<td>Gd</td>
<td>—</td>
<td>2060</td>
<td>7.63</td>
<td>292</td>
</tr>
<tr>
<td>Dy</td>
<td>—</td>
<td>2920</td>
<td>10.2</td>
<td>88</td>
</tr>
<tr>
<td>MnAs</td>
<td>670</td>
<td>870</td>
<td>3.4</td>
<td>318</td>
</tr>
<tr>
<td>MnBi</td>
<td>620</td>
<td>680</td>
<td>3.52</td>
<td>630</td>
</tr>
<tr>
<td>MnSb</td>
<td>710</td>
<td>—</td>
<td>3.5</td>
<td>587</td>
</tr>
<tr>
<td>CrO₂</td>
<td>515</td>
<td>—</td>
<td>2.03</td>
<td>386</td>
</tr>
<tr>
<td>MnOF₆₂O₃</td>
<td>410</td>
<td>—</td>
<td>5.0</td>
<td>573</td>
</tr>
<tr>
<td>FeOF₆₂O₃</td>
<td>480</td>
<td>—</td>
<td>4.1</td>
<td>858</td>
</tr>
<tr>
<td>NiOF₆₂O₃</td>
<td>270</td>
<td>—</td>
<td>2.4</td>
<td>(858)</td>
</tr>
<tr>
<td>CuOF₆₂O₃</td>
<td>135</td>
<td>—</td>
<td>1.3</td>
<td>728</td>
</tr>
<tr>
<td>MgOF₆₂O₃</td>
<td>110</td>
<td>—</td>
<td>1.1</td>
<td>713</td>
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<tr>
<td>EuO</td>
<td>—</td>
<td>1920</td>
<td>6.8</td>
<td>69</td>
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<tr>
<td>(Y₃Fe₅O₁₂)</td>
<td>130</td>
<td>200</td>
<td>5.0</td>
<td>560</td>
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</table>
Ferromagnetism

- **Curie-Weiss law**
  - Curie temperature of various ferromagnetic materials

### TABLE 3.2 Fundamental Magnetic Data for Various Crystalline Ferromagnets

<table>
<thead>
<tr>
<th>Substance</th>
<th>Structure</th>
<th>$M_s$ (290 K) (emu/cm^3)</th>
<th>$M_s$ (0 K) (emu/cm^3)</th>
<th>$n_B = \frac{M_s}{\mu_B N_v}$ ($\mu_B$)</th>
<th>$T_C(T_N)$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe</td>
<td>BCC</td>
<td>1707</td>
<td>1740</td>
<td>2.22</td>
<td>1043</td>
</tr>
<tr>
<td>Co</td>
<td>HCP, FCC</td>
<td>1440</td>
<td>1446</td>
<td>1.72</td>
<td>1388</td>
</tr>
<tr>
<td>Ni</td>
<td>FCC</td>
<td>485</td>
<td>510</td>
<td>0.606</td>
<td>627</td>
</tr>
<tr>
<td>Ni$<em>{80}$Fe$</em>{20}$</td>
<td>FCC</td>
<td>800</td>
<td>930</td>
<td>1.0</td>
<td>—</td>
</tr>
<tr>
<td>Gd</td>
<td>HCP</td>
<td>—</td>
<td>2060</td>
<td>7.63</td>
<td>292</td>
</tr>
<tr>
<td>Dy</td>
<td>HCP</td>
<td>—</td>
<td>2920</td>
<td>10.2</td>
<td>88</td>
</tr>
<tr>
<td>MnBi</td>
<td>NiAs(hex)</td>
<td>620</td>
<td>680</td>
<td>3.52</td>
<td>630</td>
</tr>
<tr>
<td>Ni$_2$MnGa</td>
<td>Heusler</td>
<td>480</td>
<td>—</td>
<td>—</td>
<td>373</td>
</tr>
<tr>
<td>Cr$_2$O$_3$</td>
<td>—</td>
<td>515</td>
<td>—</td>
<td>2.03</td>
<td>386</td>
</tr>
<tr>
<td>MnOFe$_2$O$_3$</td>
<td>Spinel</td>
<td>410</td>
<td>—</td>
<td>5.0</td>
<td>573</td>
</tr>
<tr>
<td>FeOFe$_2$O$_3$</td>
<td>Spinel</td>
<td>480</td>
<td>—</td>
<td>4.1</td>
<td>858</td>
</tr>
<tr>
<td>CoOFe$_2$O$_3$</td>
<td>Spinel</td>
<td>—</td>
<td>—</td>
<td>3.2</td>
<td>—</td>
</tr>
<tr>
<td>NiOFe$_2$O$_3$</td>
<td>Spinel</td>
<td>270</td>
<td>—</td>
<td>2.4</td>
<td>858</td>
</tr>
<tr>
<td>CuOFe$_2$O$_3$</td>
<td>Spinel</td>
<td>135</td>
<td>—</td>
<td>1.3</td>
<td>728</td>
</tr>
</tbody>
</table>

The Quantity $n_B$ is called the magneton number, the number of bohr magnetons per atom or per formula unit in a material.
Ferromagnetism

- Bloch’s law
  - Magnons / spinwaves

\[ M_s(T) = M_s(0) \left( 1 - \left( \frac{T}{T_c} \right)^{3/2} \right) \]
Interactions: From atom to atoms

- **Dipolar interaction**
  - The magnetic field induced by a magnetic dipole moment $\mu$

\[
B(\mathbf{r}) = \frac{\mu_0}{4\pi r^3} \left( \frac{3\mathbf{r}(\mathbf{\mu} \cdot \mathbf{r})}{r^2} - \mathbf{\mu} \right)
\]

- If there are two moments, then the dipolar interaction energy is

\[
E(\mathbf{r}) = \frac{\mu_0}{4\pi r^3} \left[ \frac{3(\mathbf{\mu}_1 \cdot \mathbf{r})(\mathbf{\mu}_2 \cdot \mathbf{r})}{r^2} - (\mathbf{\mu}_1 \cdot \mathbf{\mu}_2) \right] \sim 10^{-23} \text{ J}
\]

(Hw.3) $\propto \mathbf{\mu}_1 \cdot \mathbf{\mu}_2$  

(~SOI energy, Hw.2)
Interactions: From atom to atoms

- Electronic structure of atoms

Covalent, metallic bonds (delocalized)

Polar bond (localized)
Another quantum mechanics in one slide

- Linear combination of atomic orbitals (LCAO)

Keywords: Identical particles, exchange symmetry, Pauli exclusion principle, fermions
Another quantum mechanics in one slide

- **Anti-bonding orbitals**
  - Favor **ferromagnetism**
  - Antisymmetric wave fn

- **Bonding orbitals**
  - Favor **superconductivity**
  - Symmetric wave fn

Keywords: Identical particles, exchange symmetry, Pauli exclusion principle, fermions
Another quantum mechanics in one slide

- **Anti-bonding orbitals**
  - Favor **ferromagnetism**
  - Antisymmetric wave fn

\[
\Psi_A = \frac{1}{\sqrt{2}} \left[ \psi_a (r_1) \psi_b (r_2) - \psi_a (r_2) \psi_b (r_1) \right] \cdot \frac{1}{\sqrt{2}} (\uparrow\downarrow + \downarrow\uparrow)
\]

- **Bonding orbitals**
  - Favor **superconductivity**
  - Symmetric wave fn

\[
\Psi_s = \frac{1}{\sqrt{2}} \left[ \psi_a (r_1) \psi_b (r_2) + \psi_a (r_2) \psi_b (r_1) \right] \cdot \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow)
\]

Keywords: Identical particles, exchange symmetry, Pauli exclusion principle, fermions
Another quantum mechanics in one slide

- Hamiltonian

\[ \hat{H} \propto S_1 \cdot S_2 \]

\[ \frac{1}{4} \quad s = 1 \]
\[ -\frac{3}{4} \quad s = 0 \]

\[ E_S = \int \Psi_S^* \hat{H} \Psi_S \, d\mathbf{r}_1 d\mathbf{r}_2 \]

\[ E_A = \int \Psi_A^* \hat{H} \Psi_A \, d\mathbf{r}_1 d\mathbf{r}_2 \]

\[ \hat{H}_{(\text{eff})} \equiv \frac{1}{4} \left( E_S + 3E_A \right) - \left( E_S - E_A \right) S_1 \cdot S_2 \]

\[ E_S - E_A = 2 \int \psi_a^* (\mathbf{r}_1) \psi_b^* (\mathbf{r}_2) \hat{H} \psi_a (\mathbf{r}_2) \psi_b (\mathbf{r}_1) \, d\mathbf{r}_1 d\mathbf{r}_2 \]

(Why? Hw.3)
Another quantum mechanics in one slide

- Exchange integral $J$

\[ J \equiv \frac{E_S - E_A}{2} = \int \psi_a^*(\mathbf{r}_1) \psi_b^*(\mathbf{r}_2) \hat{H} \psi_a(\mathbf{r}_2) \psi_b(\mathbf{r}_1) d\mathbf{r}_1 d\mathbf{r}_2 \]

\[ \hat{H}_{(\text{eff})} = \frac{1}{4} (E_S + 3E_A) - (E_S - E_A) \mathbf{S}_1 \cdot \mathbf{S}_2 \]

\[ \hat{H}_{\text{spin}} = -(E_S - E_A) \mathbf{S}_1 \cdot \mathbf{S}_2 = -2J \mathbf{S}_1 \cdot \mathbf{S}_2 \]

\[ J > 0 \iff E_S > E_A \iff s = 1 \]

\[ J < 0 \iff E_S < E_A \iff s = 0 \]
Exchange Interaction Hamiltonian

- Heisenberg model

  If more than two spins (and let us just consider spin-1/2)...

\[ \hat{H} = -\sum_{i,j} J_{ij} S_i \cdot S_j \]

\[ J \sim 10^{-21} \text{J} \]

(Hw.3)

\[ \hat{H} = -2\sum_{i<j} J_{ij} S_i \cdot S_j \]

Note that the \( S \) here is unitless (in previous classes we used them with unit of \( \hbar \))
Molecular field theory re-visit

- Assumed that $J$ is the same for each nearest-neighbor pair

  For a particular atom $i$

\[
E_{ex}^i = \langle \hat{H}^i \rangle = -2JS_i \cdot \sum_j S_j = -\mu_i \cdot B_{mf} = -g\mu_B S_i \cdot \mu_0 H_{mf}
\]

\[
H_{mf} = \frac{2J}{\mu_0 g \mu_B} \sum_j S_j \approx \frac{2zJ}{\mu_0 g \mu_B} \langle S_j \rangle
\]  
  (sum over $z$ nearest neighbors)

\[
H_{mf} = \begin{bmatrix}
  \frac{2zJ}{\mu_0 n_v (g \mu_B)^2}
\end{bmatrix} M = \lambda M
\]

\[
M = n_v g \mu_B \langle m_j \rangle
\]
Molecular field theory re-visit

- Curie temperature

\[
\lambda = \frac{2zJ}{\mu_0 n_v (g \mu_B)^2}
\]

\[
T_c = \lambda C = \frac{\lambda n_v \mu_0 \mu_{\text{eff}}^2}{3k_B} = \frac{2zJ n_v \mu_0 \mu_{\text{eff}}^2}{\chi_0 \chi_v (g \mu_B)^2 3k_B}
\]

\[
T_c = \frac{2zJ \mu_{\text{eff}}^2}{(g \mu_B)^2 3k_B} = \frac{2zJ \cdot s(s+1)}{3k_B}
\]
Molecular field theory re-visit

- Curie temperature (general form)

\[
\lambda = \frac{2zJ \cdot (g_j - 1)^2}{\mu_0 n_v (g \mu_B)^2}
\]

\[
T_c = \frac{2zJ \cdot (g_j - 1)^2 \cdot j(j+1)}{3k_B}
\]

(general form)

\[
G = j(j+1)(g_j - 1)^2 \quad \text{De Gennes factor}
\]
Exchange stiffness constant

- From a semi-classical approach

\[ E_{ex} = \langle \hat{H} \rangle = -2 \sum_{i<j} J_{ij} S^2 \cos \theta_{ij} \approx J S^2 \sum_{i<j} \theta_{ij}^2 + \text{const.} \]

\[ \frac{E_{ex}^{ij}}{V} \approx \frac{JS^2}{V} \left( a \frac{\partial \theta_{ij}}{\partial x_{ij}} \right)^2 = A \left( \frac{\partial \theta_{ij}}{\partial x_{ij}} \right)^2 = A \left( \frac{\nabla M}{M} \right)^2 \]

Exchange stiffness constant

\[ A = \frac{2zJs^2}{a} \sim 10^{-11} \text{ J/m} \]

\[ \Delta \approx (\pi) \sqrt{A/K_{eff}} \]

(domain wall width/thickness)
Category of exchange interactions

Hierarchy of exchange coupling.

- **P**: The Pauli exclusion principle is the basis of all exchange forces.
- **E**: An exchange interaction is a metaphorical description of the effects of the Pauli principle on the Coulomb repulsion between fermions.
- **R**: RKKY is an indirect exchange where itinerant electrons are the intermediaries.
- **S**: Superexchange is an indirect exchange where the intermediary is a ligand.
- **D**: Direct exchange is a coupling between quantum systems close enough to have overlapping wave functions.
- **DM₁**: The Dzyaloshinsky-Moriya coupling occurs when the spin information between the indirectly coupled systems is upset asymmetrically by spin–orbit effects. In this version, itinerant electrons are the intermediaries.
- **DM₂**: As in DM₁ except in this version the spin–orbit coupling occurs at an intermediate ligand.

C. M. Hurd, Contemporary Physics, 23:5, 469 (1982)
Category of exchange interactions

• Direct exchange
  – In fact this only plays a minor role in determination of magnetic properties
  – Extremely small effect

• Indirect exchange
  – Superexchange (oxides)
  – Ruderman-Kittel-Kasuya-Yoshida interaction, RKKY (metals)

• Anisotropic exchange
  – Dzyaloshinskii-Moriya interaction (DMI)
  – DMI and its correlated spintexture (chiral domain walls, skyrmions) are still hot topics in the spintronics community
Superexchange

- Superexchange
  - Indirect exchange
  - a.k.a Kramers–Anderson superexchange
  - Goodenough-Kanamori rules

- Example: MnO (Manganese oxide, antiferromagnetic)
Superexchange

- **Example: MnO** (Manganese oxide, antiferromagnetic)
  - Rocksalt structure
  - Antiferromagnetic
  - Goodenough-Kanamori rules
Superexchange

- Example: MnO (Manganese oxide, antiferromagnetic)
  - Rocksalt structure
  - Antiferromagnetic
  - Goodenough-Kanamori rules
Iron oxides (ferrimagnets)

- Commonly seen Fe-oxide structures
  - Rocksalt (FeO)
  - Spinel (AFe$_2$O$_4$)
  - Garnet (A$_3$Fe$_5$O$_{12}$)
  - Perovskite (AFeO$_3$)

- Structure and magnetic properties of iron oxides

<table>
<thead>
<tr>
<th>Iron Oxide</th>
<th>Structure</th>
<th>Magnetic Structure</th>
<th>$\mu_m$(µ$_B$)</th>
<th>$\sigma$(emu/g)</th>
<th>$T_N$(K)</th>
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<tbody>
<tr>
<td>FeO</td>
<td>Rocksalt</td>
<td>Antiferromagnetic</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$-Fe$_2$O$_3$</td>
<td>Metastable</td>
<td>Ferrimagnetic</td>
<td>5.0</td>
<td>74</td>
<td>863–945</td>
</tr>
<tr>
<td>magnetite</td>
<td>defect spinel</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FeO : Fe$_2$O$_3$</td>
<td>Spinel</td>
<td>Ferrimagnetic</td>
<td>4.1</td>
<td>84</td>
<td>850</td>
</tr>
<tr>
<td>magnetite</td>
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<tr>
<td>$\alpha$-Fe$_2$O$_3$</td>
<td>Corundum (hexagonal)</td>
<td>Antiferromagnetic</td>
<td>5</td>
<td>0</td>
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<tr>
<td>hematite</td>
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</table>
Spinel ferrite

- Spinel structure

Note that antiferromagnetic coupling between A and B sites

$\text{A} - \text{O} - \text{B} \quad 125^\circ$

$\text{B} - \text{O} - \text{B} \quad 90^\circ$

$\text{A} - \text{O} - \text{A} \quad \text{nonexist}$
Spinel ferrite

- Spinel structure

![Spinel structure diagram]

Figure 4.6 Spinel structure showing transition metal sites that are octahedrally coordinated by oxygen anions. The shaded oxygen atom is shown in two subcells; it links A and B sites.

- Possible distributions of Fe ions

<table>
<thead>
<tr>
<th></th>
<th>&quot;Inverse&quot;</th>
<th></th>
<th>&quot;Normal&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A (tetrahedral)</td>
<td>B (octahedral)</td>
<td>A (tetrahedral)</td>
</tr>
<tr>
<td></td>
<td>Fe(^{3+} + 5\mu_B)</td>
<td>Fe(^{2+} + Fe^{3+} -(4+5)\mu_B)</td>
<td>Fe(^{2+} + 4\mu_B)</td>
</tr>
</tbody>
</table>

*aThe observed magnetic moment of 4\mu_B/FU suggests the “inverse” occupation applies.*
T-dependence of $M$ in ferrimagnets

- Competition between moments of two sublattices

Example: Iron garnets

Fig. 9 Experimental values of the saturation magnetization versus temperature of various iron garnets. The formula unit is $M_3Fe_5O_{12}$, where $M$ is a trivalent metal ion. The temperature at which the magnetization crosses zero is called the compensation temperature; here the magnetization of the $M$ sublattice is equal and opposite to the net magnetization of the ferric ion sublattices. Per formula unit there are 3 $Fe^{3+}$ ions on tetrahedral sides $d$; 2 $Fe^{3+}$ ions on octahedral sites $a$; and 3 $M^{3+}$ ions on sites denoted by $c$. The ferric ions contribute $(3 - 2)5\mu_B = 5\mu_B$ per formula unit. The ferric ion coupling is strong and determines the Curie temperature. If the $M^{3+}$ ions are rare-earth ions they are magnetized opposite to the resultant of the $Fe^{3+}$ ions. The $M^{3+}$ contribution drops rapidly with increasing temperature because the $M$-Fe coupling is weak.
T-dependence of $M$ in ferrimagnets

- Competition between moments of two sublattices

Example: NiFe$_{2-x}$V$_x$O$_4$
Néel model for ferrimagnets

• Curie-Weiss model

\[ H_E = \lambda M \quad \Rightarrow M = \chi H = \chi \left( H_{\text{applied}} + H_E \right) = \frac{C}{T} \left( H_{\text{applied}} + \lambda M \right) \]

\[ \Rightarrow \chi = \frac{M}{H_{\text{applied}}} = \frac{C}{T - \lambda C} = \frac{C}{T - T_c} \]

• Néel model

– Consider two sublattices: A sites and B sites

\[ H_E^A = \lambda_{AA} M_A - \lambda_{AB} M_B \quad \Rightarrow M_A = \frac{C_A}{T} \left( H_{\text{applied}} + \lambda_{AA} M_A - \lambda_{AB} M_B \right) \]

\[ H_E^B = \lambda_{BB} M_B - \lambda_{AB} M_A \quad \Rightarrow M_B = \frac{C_B}{T} \left( H_{\text{applied}} + \lambda_{BB} M_B - \lambda_{AB} M_A \right) \]
Néel model for ferrimagnets

- Néel model
  - Consider two sublattices: A sites and B sites

\[
M_A = \frac{C_A}{T} \left( H_{\text{applied}} - \lambda M_B \right) \quad \lambda = \lambda_{AB} \gg \lambda_{AA}
\]

\[
M_B = \frac{C_B}{T} \left( H_{\text{applied}} - \lambda M_A \right) \quad \lambda = \lambda_{AB} \gg \lambda_{BB}
\]

\[
T_c = \lambda \sqrt{C_A C_B}
\]

(Hw.3)

\[
\chi = \frac{M_A + M_B}{H_{\text{applied}}} = \frac{(C_A + C_B)T - 2\lambda C_A C_B}{T^2 - T_c^2}
\]
Néel model for ferrimagnets

- Néel model
  - Consider two sublattices: A sites and B sites

\[
\chi^{-1} = \frac{T^2 - T_c^2}{(C_A + C_B)T - 2\lambda C_A C_B}
\]
Néel model for antiferromagnets

- Néel model (antiferromagnetism)
  - Néel temperature
    \[ C = C_A = C_B \]
    \[ T_N = \lambda C \]
  - Susceptibility
    \[ \chi = \frac{2C}{T + T_N} \]
    \[ \chi = \frac{C_p}{T + \theta_p} \]
Magnetization and susceptibility revisit

- Susceptibility revisit

http://nptel.ac.in/courses/113104005/81